# Lecture 2: Construction of Suffix Arrays 

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## Taxonomy


source: Puglisi/Smyth/Turpin ACM Computing Surveys '07

## Induced Sorting

- [Nong/Zhang/Chan DCC'09] sais-algorithm:
$\checkmark O(n)$ in theory
$\checkmark$ fast in practice
$\checkmark$ as simple as Kärkkäinen/Sanders DC3


## Algorithm sais

- Definition: suffix $T[i, n]$ called
- S-type iff $T[i . . n]<\operatorname{lex} T[i+1 . . n]$ (T[n,n]='\$' always S)
- L-type otherwise
I. Choose sample: leftmost $S$ (predecessor is L ), $\left|\mathrm{S}^{*}\right|<\mathrm{I} / 2 n$

2. Sort $\mathrm{S}^{*}$-suffixes by recursion

- on new text formed by sorted $S^{*}$-substrings

3. Scan A from left to right (say we're at pos.i):

- if $T[A[i]-I]$ is $\mathbf{L}$, write $A[i]-I$ to Ist pos. in bucket

4. like (3), but sorting $\mathbf{S}$-suffixes in a right-to-left scan

- if $T[A[i]-I]$ is $\mathbf{S}$, write $A[i]$-I to last pos. in bucket

$$
T=\begin{array}{llllllllllll}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
\mathrm{c} & \mathrm{a} & \mathrm{~b} & \mathrm{c} & \mathrm{c} & \mathrm{~b} & \mathrm{a} & \mathrm{a} & \mathrm{a} & \mathrm{~b} & \mathrm{~b} & \mathrm{a} \\
\mathrm{~L} & \mathrm{~S}^{*} & \mathrm{~S} & \mathrm{~L} & \mathrm{~L} & \mathrm{~L} & \mathrm{~S}^{*} & \mathrm{~S} & \mathrm{~S} & \mathrm{~L} & \mathrm{~L} & \mathrm{~L} \\
\mathrm{~S}^{*}
\end{array}
$$

## Sorting $S^{*}$-Substrings

- Same algorithm, but with UNSORTED $S^{*}$-suffixes
I. Choose sample: leftmost $S$ (call them $S^{*}$ ), $\left|S^{*}\right|<1 / 2 n$

2. Put $S^{*}$-substrings in their buckets (in text order)
3. Scan $A$ from left to right (say we're at pos.i):

- if $T[A[i]-I]$ is $\mathbf{L}$, write $A[i]-I$ to Ist pos. in bucket

4. like (3), but sorting $\mathbf{S}$ substrings in a right-to-left scan

## Correctness

- 2 main points:
- S-substrings > L-substrings in same bucket
- order of suffixes in reduced substring $\hat{=}$ order in original string
- full proof: consult section 3.2 in:
- Ge Nong, Sen Zhang, Wai Hong Chan: Two Efficient Algorithms for Linear Time Suffix Array Construction. IEEE Trans. Computers 60(I0): I47I-I484 (201I)

